Big Ideas Math: Algebra 2

Chapter Summary

Chapter 11: Data Analysis and Statistics

Standards

Common Core:
HSS-ID.A.4, HSS-IC.A.1, HSS-IC.A.2, HSS-IC.B.3, HSS-IC.B.4, HSS-IC.B.5, HSS-IC.B.6

Learning Goals

Calculate probabilities using normal distributions.

Use *z*-scores and the standard normal table to find probabilities.

Recognize data sets that are normal.

Distinguish between populations and samples.

Analyze hypotheses.

Identify types of sampling methods in statistical studies.

Recognize bias in sampling.

Analyze methods of collecting data.

Recognize bias in survey questions.

Describe experiments.

Recognize how randomization applies to experiments and observational studies.

Analyze experimental designs.

Estimate population parameters.

Analyze estimated population parameters.

Find margins of error for surveys.

Organize data from an experiment with two samples.

Resample data using a simulation to analyze a hypothesis.

Make inferences about a treatment.

Core Vocabulary

A type of probability distribution in which the graph is a bell-shaped curve that is symmetric about the mean is a ***normal distribution.***

The graph of a normal distribution that is bell-shaped and is symmetric about the mean is a ***normal curve***.

The ***standard normal distribution*** is the normal distribution with mean 0 and standard deviation 1.

The *z*-value for a particular *x*-value is called the ***z-score*** for the *x*-value and is the number of standard deviations the *x*-value lies above or below the mean.

A ***population*** is the collection of all data, such as responses, measurements, or counts, that you want information about.

A ***sample*** is a subset of a population.

A numerical description of a population characteristic is called a ***parameter***.

A numerical description of a sample characteristic is called a ***statistic***.

In statistics, a ***hypothesis*** is a claim about a characteristic of a population.

In a ***random sample***, each member of a population has an equal chance of being selected.

For a ***self-selected sample***, members of a population can volunteer to be in the sample.

For a ***systematic sample***, a rule is used to select members of a population.

A sample in which a population is divided into smaller groups that share a similar characteristic and a sample is then randomly selected from each group is a ***stratified sample***.

A sample in which a population is divided into groups, called clusters, and all of the members in one or more of the clusters are randomly selected is a ***cluster sample***.

For a ***convenience sample***, only members of a population who are easy to reach are selected.

A ***bias*** is an error that results in a misrepresentation of a population.

An ***unbiased sample*** is representative of the population that you want information about.

A sample that overrepresents or under-represents part of the population is a ***biased sample***.

An ***experiment*** imposes a treatment on individuals in order to collect data on their response to the treatment.

An ***observational study*** observes individuals and measures variables without controlling the individuals or their environment.

A ***survey*** is an investigation of one or more characteristics of a population.

A ***simulation*** uses a model to reproduce the conditions of a situation or process so that the simulated outcomes closely match the real-world outcomes.

Questions that are flawed in a way that leads to inaccurate results are called ***biased questions***.

In a ***controlled experiment***, two groups are studied under identical conditions with the exception of one variable.

In an experiment, the ***control group*** is under ordinary conditions and is subjected to no treatment.

The group that is subjected to the treatment in an experiment is the ***treatment group***.

Randomization is a process of randomly assigning subjects to different treatment groups.

In a ***randomized comparative experiment***, subjects are randomly assigned to the control group or the treatment group.

A ***placebo*** is a harmless, unmedicated treatment that resembles the actual treatment.

The repetition of an experiment under the same or similar conditions to improve the validity of the experiment is called ***replication.***

Descriptive statistics is the branch of statistics that involves the organization, summarization, and display of data**.**

Inferential statistics is the branch of statistics that involves using a sample to draw conclusions about a population.

The limit on how much the responses of the sample would differ from the responses of the population is the ***margin of error***.

Core Concept

Areas Under a Normal Curve

A normal distribution with mean μ and standard deviation σ has these properties.

- The total area under the related normal curve is 1.
- About 68% of the area lies within 1 standard deviation of the mean.
- About 95% of the area lies within 2 standard deviations of the mean.
- About 99.7% of the area lies within 3 standard deviations of the mean.

Comparative Studies and Causality

- A rigorous randomized comparative experiment, by eliminating sources of variation other than the controlled variable, can make valid cause-and-effect conclusions possible.
- An observational study can identify *correlation* between variables, but not *causality*. Variables, other than what is being measured, may be affecting the results.

Types of Samples

- For a self-selected sample, members of a population can volunteer to be in the sample.
- For a systematic sample, a rule is used to select members of a population. For instance, selecting every other person.
- For a stratified sample, a population is divided into smaller groups that share a similar characteristic. A sample is then randomly selected from each group.
- For a cluster sample, a population is divided into groups, called *clusters*. All of the members in one or more of the clusters are selected.
- For a convenience sample, only members of a population who are easy to reach are selected.

Methods of Collecting Data

- An experiment imposes a treatment on individuals in order to collect data on their response to the treatment. The treatment may be a medical treatment, or it can be any action that might affect a variable in the experiment, such as adding methanol to gasoline and then measuring its effect on fuel efficiency.
- An observational study observes individuals and measures variables without controlling the individuals or their environment. This type of study is used when it is difficult to control or isolate the variable being studied, or when it may be unethical to subject people to a certain treatment or to withhold it from them.
- A survey is an investigation of one or more characteristics of a population. In a survey, every member of a sample is asked one or more questions.
- A simulation uses a model to reproduce the conditions of a situation or process so that the simulated outcomes closely match the real-world outcomes. Simulations allow you to study situations that are impractical or dangerous to create in real life.

Margin of Error Formula

When a random sample of size n is taken from a large population, the margin of error is approximated by

$$\text{Margin of error} = \pm\frac{1}{\sqrt{n}}.$$

This means that if the percent of the sample responding a certain way is p (expressed as a decimal), then the percent of the population who would respond the same way is likely to be between $p - \frac{1}{\sqrt{n}}$ and $p + \frac{1}{\sqrt{n}}$.

Essential Questions

In a normal distribution, about what percent of the data lies within one, two, and three standard deviations of the mean?

How can you test theoretical probability using sample data?

What are some considerations when undertaking a statistical study?

How can you use an experiment to test a conjecture?

How can you use a sample survey to infer a conclusion about a population?

How can you test a hypothesis about an experiment?

Game

- M and M and M

This is available online in the *Game Closet* at www.bigideasmath.com.

Additional Review

- Using *z*-Scores and the Standard Normal Table, *p. 597*
- Recognizing Normal Distributions, *p. 599*
- Distinguishing Between Populations and Samples, *p. 604*
- Analyzing Hypotheses, *p. 606*
- Randomization in Experiments and Observational Studies, *p. 621*
- Analyzing Experimental Designs, *p. 622*
- Estimating Population Parameters, *p. 626*
- Analyzing Estimated Population Parameters, *p. 628*
- Experiments with Two Samples, *p. 634*
- Resampling Data Using Simulations, *p. 635*
- Making Inferences About Treatments, *p. 636*

What's the Point?

The STEM Videos available online show ways to use mathematics in real-life situations.

The Chapter 11: Volcano Damage STEM Video is available online at www.bigideasmath.com.