

# Chapter Summary

## Chapter 12: Probability

### Games

- Anything but Eight
- Take Your Chances
- War with a Twist

These are available online in the *Game Closet* at [www.bigideasmath.com](http://www.bigideasmath.com).

### Standards

Common Core:  
HSA-APR.C.5, HSS-CP.A.1, HSS-CP.A.2,  
HSS-CP.A.3, HSS-CP.A.4, HSS-CP.A.5,  
HSS-CP.B.6, HSS-CP.B.7, HSS-CP.B.8,  
HSS-CP.B.9

### Learning Goals

Find sample spaces.

Find theoretical probabilities.

Find experimental probabilities.

Determine whether events are independent events.

Find probabilities of independent and dependent events.

Find conditional probabilities.

Make two-way tables.

Find relative and conditional relative frequencies.

Use conditional relative frequencies to find conditional probabilities.

Find probabilities of compound events.

Use more than one probability rule to solve real-life problems.

Use the formula for the number of permutations.

Use the formula for the number of combinations.

Use combinations and the Binomial Theorem to expand binomials.

Construct and interpret probability distributions.

Construct and interpret binomial distributions.

### Core Vocabulary

A **probability experiment** is an action, or trial, that has varying results.

The possible results of a probability experiment are **outcomes**.

A collection of one or more outcomes in a probability experiment is an **event**.

The set of all possible outcomes for an experiment is called a **sample space**.

The **probability of an event** is a measure of the likelihood, or chance, that the event will occur.

The ratio of the number of favorable outcomes to the total number of outcomes when all outcomes are equally likely is the **theoretical probability** of the event.

A probability found by calculating a ratio of two lengths, areas, or volumes is called **geometric probability**.

An **experimental probability** is the ratio of the number of successes, or favorable outcomes, to the number of trials in a probability experiment.

Two events are **independent events** when the occurrence of one event does not affect the occurrence of the other event.

Two events are **dependent events** when the occurrence of one event *does* affect the occurrence of the other event.

The probability that event  $B$  occurs given that event  $A$  has occurred is called the **conditional probability** of  $B$  given  $A$  and is written as  $P(B | A)$ .

A **two-way table** is a frequency table that displays data collected from one source that belong to two different categories.

Each entry in a two-way table is called a **joint frequency**.

The sums of the rows and columns in a two-row table are called **marginal frequencies**.

A **joint relative frequency** is the ratio of a frequency that is not in the total row or the total column of the total number of values or observations in a two-way table.

A **marginal relative frequency** is the sum of the joint relative frequencies in a row or a column in a two-way table.

A **conditional relative frequency** is the ratio of a joint relative frequency to the marginal relative frequency in a two-way table.

The union or intersection of two events is called a **compound event**.

Two events that have one or more outcomes in common are called **overlapping events**.

Two events are **disjoint**, or **mutually exclusive**, when they have no outcomes in common.

A **permutation** is an arrangement of objects in which order is important.

The product of the integers from 1 to  $n$ , for any positive integer  $n$  is called  **$n$  factorial**.

A **combination** is a selection of objects in which order is *not* important.

The **Binomial Theorem** states that for any positive integer  $n$ , the binomial expansion of  $(a + b)^n$  is  $(a + b)^n = {}_nC_0a^n b^0 + {}_nC_1a^{n-1}b^1 + {}_nC_2a^{n-2}b^2 + \dots + {}_nC_na^0b^n$ .

A **random variable** is a variable whose value is determined by the outcomes of a probability experiment.

A **probability distribution** is a function that gives the probability of each possible value of a random variable.

A type of probability distribution that shows the probabilities of the outcomes of a binomial experiment is a **binomial distribution**.

An experiment in which there are a fixed number of independent trials, exactly two possible outcomes for each trial, and the probability of success is the same for each trial is a **binomial experiment**.

## Core Concept

### Probability of the Complement of an Event

The probability of the complement of event  $A$  is  

$$P(\bar{A}) = 1 - P(A).$$

### Probability of Independent Events

- Two events  $A$  and  $B$  are independent events if and only if the probability that both events occur is the product of the probabilities of the events.
- $P(A \text{ and } B) = P(A) \cdot P(B)$

### Probability of Dependent Events

- If two events  $A$  and  $B$  are dependent events, then the probability that both events occur is the product of the probability of the first event and the conditional probability of the second event given the first event.
- $P(A \text{ and } B) = P(A) \cdot P(B | A)$

### Probability of Compound Events

- If  $A$  and  $B$  are any two events, then the probability of  $A$  or  $B$  is  

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$
- If  $A$  and  $B$  are disjoint events, then the probability of  $A$  or  $B$  is  

$$P(A \text{ or } B) = P(A) + P(B).$$

### Probability Distributions

- A probability distribution is a function that gives the probability of each possible value of a random variable.
- The sum of all the probabilities in a probability distribution must equal 1.

### The Binomial Theorem

For any positive integer  $n$ , the binomial expansion of  $(a + b)^n$  is

$$(a + b)^n = {}_nC_0 a^n b^0 + {}_nC_1 a^{n-1} b^1 + {}_nC_2 a^{n-2} b^2 + \dots + {}_nC_n a^0 b^n.$$

Notice that each term in the expansion of  $(a + b)^n$  has the form  ${}_nC_r a^{n-r} b^r$ , where  $r$  is an integer from 0 to  $n$ .

### Relative and Conditional Relative Frequencies

- A joint relative frequency is the ratio of a frequency that is not in the total row or the total column to the total number of values or observations.
- A marginal relative frequency is the sum of the joint relative frequencies in a row or a column.
- A conditional relative frequency is the ratio of a joint relative frequency to the marginal relative frequency.
- You can find a conditional relative frequency using a row total or a column total of a two-way table.

### Permutations

- The number of permutations of  $n$  objects is given by  

$${}_nP_n = n!.$$
- The number of permutations of  $n$  objects taken  $r$  at a time, where  $r \leq n$ , is given by  ${}_nP_r = \frac{n!}{(n-r)!}$ .

### Combinations

- The number of combinations of  $n$  objects taken  $r$  at a time, where  $r \leq n$ , is given by  ${}_nC_r = \frac{n!}{(n-r)! \cdot r!}$ .

### Binomial Experiments

A binomial experiment meets the following conditions.

- There are  $n$  independent trials.
- Each trial has only two possible outcomes: success and failure.
- The probability of success is the same for each trial. This probability is denoted by  $p$ . The probability of failure is  $1 - p$ .

For a binomial experiment, the probability of exactly  $k$  successes in  $n$  trials is  $P(k \text{ successes}) = {}_nC_k p^k (1 - p)^{n-k}$ .

## Essential Questions

How can you list the possible outcomes in the sample space of an experiment?

How can you determine whether two events are independent or dependent?

How can you construct and interpret a two-way table?

How can you find probabilities of disjoint and overlapping events?

How can a tree diagram help you visualize the number of ways in which two or more events can occur?

How can you determine the frequency of each outcome of an event?

## Additional Review

- Theoretical Probabilities, p. 668
- Experimental Probabilities, p. 671
- Finding Conditional Probabilities, p. 679
- Making Two-Way Tables, p. 684

## What's the Point?

The STEM Videos available online show ways to use mathematics in real-life situations.

The Chapter 12: Arborists STEM Video is available online at [www.bigideasmath.com](http://www.bigideasmath.com).