

# Chapter Summary

## Chapter 7: Rational Functions

### Learning Goals

Classify direct and inverse variation.

Write inverse variation equations.

Graph simple rational functions.

Translate simple rational functions.

Graph other rational functions.

Simplify rational expressions.

Multiply rational expressions.

Divide rational expressions.

Add or subtract rational expressions.

Rewrite rational expressions and graph the related function.

Simplify complex fractions.

Solve rational equations by cross multiplying.

Solve rational equations by using the least common denominator.

Use inverses of functions.

### Core Vocabulary

Two variables  $x$  and  $y$  show **inverse variation** when  $y = \frac{a}{x}$ , where  $a \neq 0$ .

The constant  $a$  in the inverse variation equation  $y = \frac{a}{x}$ , where  $a \neq 0$  is the **constant of variation**.

A **rational function** has the form  $f(x) = \frac{p(x)}{q(x)}$ , where  $p(x)$  and  $q(x)$  are polynomials and  $q(x) \neq 0$ .

A **rational expression** is a fraction whose numerator and denominator are nonzero polynomials.

A rational expression is in **simplified form** when its numerator and denominator have no common factors (other than  $\pm 1$ ).

A **complex fraction** is a fraction that contains a fraction in its numerator or denominator.

**Cross multiplying** is a method used to solve a rational equation when each side of the equation is a single rational expression.

### Standards

Common Core:

HSA-CED.A.1, HSA-CED.A.2, HSA-CED.A.3, HSA-CED.A.4, HSA-APR.D.6, HSA-APR.D.7, HSA-REI.A.1, HSA-REI.A.2, HSF-BF.B.3

### Essential Questions

How can you recognize when two quantities vary directly or inversely?

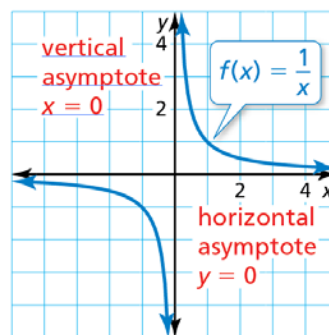
What are some of the characteristics of the graph of a rational function?

How can you determine the excluded values in a product or quotient of two rational expressions?

How can you determine the domain of the sum or difference of two rational expressions?

How can you solve a rational equation?

### Core Concept



### Parent Function for Simple Rational Functions

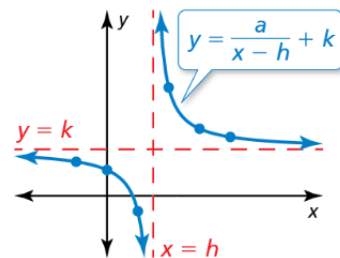
- The graph of the parent function  $f(x) = \frac{1}{x}$  is a **hyperbola**, which consists of two symmetrical parts called branches.
- The domain and range are all nonzero real numbers.
- Any function of the form  $g(x) = \frac{a}{x}$  ( $a \neq 0$ ) has the same asymptotes, domain, and range as the function  $f(x) = \frac{1}{x}$ .

## Core Concept

### Graphing Translations of Simple Rational Functions

To graph a rational function of the form  $y = \frac{a}{x-h} + k$ , follow these steps:

- Step 1** Draw the asymptotes  $x = h$  and  $y = k$ .
- Step 2** Plot points to the left and to the right of the vertical asymptote.
- Step 3** Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.



### Simplifying Rational Expressions

Let  $a$ ,  $b$ , and  $c$  be expressions with  $b \neq 0$  and  $c \neq 0$ .

Property  $\frac{ac}{bc} = \frac{a}{b}$  Divide out common factor  $c$ .

### Multiplying Rational Expressions

Let  $a$ ,  $b$ ,  $c$ , and  $d$  be expressions with  $b \neq 0$  and  $d \neq 0$ .

Property  $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$  Simplify  $\frac{ac}{bd}$  if possible.

### Dividing Rational Expressions

Let  $a$ ,  $b$ ,  $c$ , and  $d$  be expressions with  $b \neq 0$ ,  $c \neq 0$ , and  $d \neq 0$ .

Property  $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$  Simplify  $\frac{ad}{bc}$  if possible.

### Adding or Subtracting with Like Denominators

Let  $a$ ,  $b$ , and  $c$  be expressions with  $c \neq 0$ .

**Addition**  
 $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

**Subtraction**  
 $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$

### Adding or Subtracting with Unlike Denominators

Let  $a$ ,  $b$ ,  $c$ , and  $d$  be expressions with  $c \neq 0$  and  $d \neq 0$ .

**Addition**  
 $\frac{a}{c} + \frac{b}{d} = \frac{ad}{cd} + \frac{bc}{cd} = \frac{ad+bc}{cd}$

**Subtraction**  
 $\frac{a}{c} - \frac{b}{d} = \frac{ad}{cd} - \frac{bc}{cd} = \frac{ad-bc}{cd}$

### Inverse Variation

- Two variables  $x$  and  $y$  show inverse variation when they are related as follows:  $y = \frac{a}{x}$ ,  $a \neq 0$ .
- The constant  $a$  is the constant of variation, and  $y$  is said to *vary inversely* with  $x$ .

### Simplifying Complex Fractions

**Method 1** If necessary, simplify the numerator and denominator by writing each as a single fraction. Then divide by multiplying the numerator by the reciprocal of the denominator.

**Method 2** Multiply the numerator and the denominator by the LCD of every fraction in the numerator and denominator. Then simplify.

### Game

- Four in a Row
- Make My Team
- A Trick for You
- Equation Tic-Tac-Toe

These are available online in the *Game Closet* at [www.bigideasmath.com](http://www.bigideasmath.com).

### Additional Review

- Writing Inverse Variation Equations, p. 361
- Solving Rational Equations by Cross Multiplying, p. 392
- Solving Rational Equations by Using the Least Common Denominator, p. 393
- Using Inverses of Functions, p. 395

### What's the Point?

The STEM Videos available online show ways to use mathematics in real-life situations.

The Chapter 7: 3D Printing STEM Video is available online at [www.bigideasmath.com](http://www.bigideasmath.com).