Big Ideas Math: Algebra 2

Chapter Summary

Chapter 8: Sequences and Series

Core Vocabulary

A ***sequence*** is an ordered list of numbers.

The values in the range are called the ***terms of the sequence***.

When the terms of a sequence are added together, the resulting expression is a ***series***.

For any sequence a_1, a_2, $a3$, . . ., the sum of the first k terms may be written as
$\sum_{n=1}^{k} a_n = a_1 + a_2 + a_3 + \cdots + a_k$,
where k is an integer, which is called ***summation notation*** or ***sigma notation*** .

An ***arithmetic sequence*** is a sequence in which the difference of consecutive terms is constant.

The constant difference, d, between consecutive terms of an arithmetic sequence is called the ***common difference***.

The expression formed by adding the terms of an arithmetic sequence is called an ***arithmetic series***.

A ***geometric sequence*** is a sequence in which the ratio of any term to the previous term is constant.

The constant ratio, r, between consecutive terms of a geometric sequence is called the ***common ratio***.

The expression formed by adding the terms of a geometric sequence is called a ***geometric series***.

The sum S_n of the first n terms of an infinite series is called a ***partial sum***.

An ***explicit rule*** gives a_n as a function of the term's position number n in the sequence.

A ***recursive rule*** gives the beginning term(s) of a sequence and a recursive equation that tells how a_n is related to one or more preceding terms.

Learning Goals

Use sequence notation to write terms of sequence.

Write a rule for the nth term of a sequence.

Sum the terms of a sequence to obtain a series and use summation notation.

Identify arithmetic sequences.

Write rules for arithmetic sequences.

Find sums of finite arithmetic series.

Identify geometric sequences.

Write rules for geometric sequences.

Find sums of finite geometric series.

Find partial sums of infinite geometric series.

Find sums of infinite geometric series.

Evaluate recursive rules for sequences.

Write recursive rules for sequences.

Translate between recursive and explicit rules for sequences.

Use recursive rules to solve real-life problems.

Standards

Common Core:
HSA-SSE.B.4, HSF-IF.A.3, HSF-BF.A.1a, HSF-BF.A.2, HSF-LE.A.2

Essential Questions

How can you write a rule for the nth term of a sequence?

How can you recognize an arithmetic sequence from its graph?

How can you recognize a geometric sequence from its graph?

How can you find the sum of an infinite geometric series?

How can you define a sequence recursively?

Core Concept

Rule for an Arithmetic Sequence
Algebra The nth term of an arithmetic sequence with first term a_1 and common difference d is given by:

$$a_n = a_1 + (n - 1)d$$

The Sum of a Finite Arithmetic Series
The sum of the first n terms of an arithmetic series is

$$S_n = n\left(\frac{a_1 + a_n}{2}\right).$$

In words, S_n is the mean of the first and nth terms, multiplied by the number of terms.

Rule for a Geometric Sequence
Algebra The nth term of a geometric sequence with first term a_1 and common ratio r is given by:

$$a_n = a_1 r^{n-1}$$

Core Concept

Sequences

A sequence is an ordered list of numbers. A *finite sequence* is a function that has a limited number of terms and whose domain is the finite set {1, 2, 3, . . . , n}. The values in the range are called the terms of the sequence.

Domain:	1	2	3	4 . . . n	Relative position of each term
	↓	↓	↓	↓ ↓	
Range:	a_1	a_2	a_3	$a_4 \dots a_n$	Terms of the sequence

An *infinite sequence* is a function that continues without stopping and whose domain is the set of positive integers. Here are examples of a finite sequence and an infinite sequence.

Finite sequence: 2, 4, 6, 8 **Infinite sequence:** 2, 4, 6, 8, . . .

A sequence can be specified by an equation, or *rule*. For example, both sequences above can be described by the rule $a_n = 2n$ or $f(n) = 2n$.

Series and Summation Notation

When the terms of a sequence are added together, the resulting expression is a series. A series can be finite or infinite.

Finite series: 2 + 4 + 6 + 8 **Infinite series:** 2 + 4 + 6 + 8 + . . .

You can use summation notation to write a series. For example, the two series above can be written in summation notation as follows:

Finite series: $2 + 4 + 6 + 8 = \sum_{i=1}^{\infty} 2i$

Infinite series: $2 + 4 + 6 + 8 + \dots = \sum_{i=1}^{\infty} 2i$

For both series, the *index of summation* is i and the *lower limit of summation* is 1. The *upper limit of summation* is 4 for the finite series and ∞ (infinity) for the infinite series. Summation notation is also called sigma notation because it uses the uppercase Greek letter *sigma*, written Σ.

The Sum of a Finite Geometric Series

The sum of the first n terms of a geometric series with common ratio $r \neq 1$ is

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right).$$

Formulas for Special Series

Sum of n terms of 1:

$$\sum_{i=1}^{n} 1 = n$$

Sum of first n positive integers:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Sum of squares of first n positive integers:

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

The Sum of an Infinite Geometric Series

The sum of an infinite geometric series with first term a_1 and common ratio r is given by

$$S = \frac{a_1}{1-r}$$

provided $|r| < 1$. If $|r| \geq 1$, then the series has no sum.

Recursive Equations for Arithmetic and Geometric Sequences

Arithmetic Sequence
$a_n = a_{n-1} + d$, where d is the common difference

Geometric Sequence
$a_n = r \cdot a_{n-1}$, where r is the common ratio

Additional Review

- Partial Sums of Infinite Geometric Series, *p. 436*
- Evaluating Recursive Rules, *p. 442*
- Translating Between Recursive and Explicit Rules, *p. 444*

What's the Point?

The STEM Videos available online show ways to use mathematics in real-life situations.

The Chapter 8: LA River Ecology STEM Video is available online at www.bigideasmath.com.